

# Characterization of diagonally dominant H-matrices

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## Abstract

We first show that sufficient conditions for a diagonally dominant matrix to be a nonsingular one (and also an H-matrix), obtained independently by Shivakumar and Chew in 1974, and Farid in 1995, are equivalent. Then we simplify the characterization of diagonally dominant H-matrices obtained by Huang in 1995, and using it prove that the Shivakumar-Chew-Farid sufficient condition for a diagonally dominant matrix to be an H-matrix, is also necessary.

**Key words.** diagonal dominance, nonsingularity results, H-matrices, nonzero elements chain.

**AMS subject classifications.** 65F15, 65F99.

## 1 Introduction and notation

Throughout the paper, we use the following notation:

$N := \{1, 2, \dots, n\}$ , the set of all indices,

$\bar{S} := N \setminus S$ , the complement of  $S$ ,

$r_i(A) := \sum_{j \in N \setminus \{i\}} |a_{ij}|$ , deleted  $i$ th row sum,

$r_i^S(A) := \sum_{j \in S \setminus \{i\}} |a_{ij}|$ , part of the previous sum, which corresponds to the set  $S$ .

Obviously, for arbitrary set  $S \in \mathcal{P}(N) \setminus \{\emptyset, N\}$  and for each index  $i \in N$ , we have

$$r_i(A) = r_i^S(A) + r_i^{\bar{S}}(A).$$

We say that a matrix  $A \in \mathbb{C}^{n \times n}$ ,  $n \geq 2$ , is SDD (strictly diagonally dominant) if

$$|a_{ii}| > r_i(A) \text{ for all } i \in N,$$

that it is DD (diagonally dominant) if

$$|a_{ii}| \geq r_i(A) \text{ for all } i \in N,$$

and that it is DD+ if it is DD and exists  $i \in N$  such that  $|a_{ii}| > r_i(A)$ .

Let  $T(A)$  be the set of indices of non SDD rows of a matrix  $A$ ,

$$T(A) := \left\{ i \in N \mid |a_{ii}| \leq r_i(A) \right\}.$$

The following sufficient condition for a diagonally dominant matrix to be nonsingular is obtained by Shivakumar and Chew in 1974 ([1]).

**Theorem 1** *Let  $A = [a_{ij}] \in \mathbb{C}^{n \times n}$ ,  $n \geq 2$ , be a DD matrix such that  $T(A) = \emptyset$ , or for each  $i_0 \in T(A)$  there exists a nonzero elements chain of the form  $a_{i_0 i_1}, a_{i_1 i_2}, \dots, a_{i_{r-1} i_r}$ , with  $i_r \in \overline{T(A)}$ . Then  $A$  is nonsingular.*

The *nonzero elements chains* of  $A$ , mean that from every  $i \in T(A)$  there exists a path to some  $j \in \overline{T(A)}$  in the directed graph  $G(A)$ , associated to the matrix  $A$  (see [2]).

Before stating Farid's result, we need the following definition.

**Definition 1** *Let  $A = [a_{ij}] \in \mathbb{C}^{n \times n}$ ,  $n \geq 2$ , and let  $S$  be a proper subset of  $N$ . We say that the set  $S$  is interwoven for the matrix  $A$  if  $|S| \leq 1$ , or  $|S| = s > 1$  and there exist different numbers  $p_1, p_2, \dots, p_{s-1} \in S$ , as well as numbers  $q_1, q_2, \dots, q_{s-1}$  (not obligatory different), such that  $q_1 \in \overline{S}$ ,  $a_{p_1 q_1} \neq 0$  and  $q_i \in \overline{S} \cup \{p_1, p_2, \dots, p_{i-1}\}$ ,  $a_{p_i q_i} \neq 0$ , for every  $i \in \{2, 3, \dots, s-1\}$ .*

Farid obtained the following sufficient condition for a diagonally dominant matrix to be nonsingular in 1995 ([3]).

**Theorem 2** *Let  $A = [a_{ij}] \in \mathbb{C}^{n \times n}$ ,  $n \geq 2$ , be a DD matrix with nonzero diagonal entries, such that  $T(A)$  is an interwoven set of indices for  $A$ . Then  $A$  is nonsingular.*

It can easily be shown that if a matrix  $A$  satisfies conditions of Theorem 1 or 2, then it is also an H-matrix.

## 2 Characterization of diagonally dominant H-matrices

In the following theorem, we prove that sufficient conditions for a diagonally dominant matrix to be a nonsingular one (and also an H-matrix) obtained by Shivakumar and Chew in 1974, and Farid in 1995, are equivalent.

**Theorem 3** *Let  $A = [a_{ij}] \in \mathbb{C}^{n \times n}$ ,  $n \geq 2$  be a DD matrix. Then the following conditions are equivalent:*

1.  $T(A) = \emptyset$  or for each  $i_0 \in T(A)$  there exists a nonzero elements chain of the form  $a_{i_0 i_1}, a_{i_1 i_2}, \dots, a_{i_{r-1} i_r}$ , with  $i_r \in \overline{T(A)}$ ,
2. The matrix  $A$  has nonzero diagonal entries and  $T(A)$  is an interwoven set of indices for  $A$ .

*Proof.* Let us assume that the first condition is satisfied. Since  $A$  is a DD matrix, it can easily be shown that then  $A$  has nonzero diagonal entries. If  $|T(A)| \leq 1$ , then the statement holds trivially, so let us assume that  $|T(A)| = t > 1$ . Let  $i \in T(A)$  be such that the shortest path in  $G(A)$  from  $i$  to some  $j \in \overline{T(A)}$  is of length  $l$  (with  $l$  being maximal with such property). Let us put all  $t$  indices from  $T(A)$  in  $l$  sets  $N_i, i \in \{1, 2, \dots, l\}$ . We put in  $N_1$  those indices for which the shortest path to some  $j \in \overline{T(A)}$  is of length 1, in  $N_2$  those for which such path is of length 2, and so on. Numbers  $p_1, p_2, \dots, p_{t-1} \in T(A)$  are chosen in such way that  $\{p_1, \dots, p_{k_1}\} = N_1, \{p_{k_1+1}, \dots, p_{k_2}\} = N_2, \dots, \{p_{k_{m-1}+1}, \dots, p_{t-1}\} \subseteq N_m, m \in \{l-1, l\}$ . For every  $p_i \in T(A)$ , we choose an arbitrary shortest path to some  $j \in \overline{T(A)}$  and then choose  $q_i$  to be the first index after  $p_i$  on that path. It can now easily be shown with the given choice of numbers  $p_i, q_i, i \in \{1, 2, \dots, t-1\}$ , that the set  $T(A)$  is interwoven for the matrix  $A$ .

Let us now assume that the second condition is satisfied. Then  $A$  has nonzero diagonal entries. If  $|T(A)| \leq 1$ , the statement trivially holds, so let us assume that  $|T(A)| = t > 1$ . By assumption, there exist different numbers  $p_1, p_2, \dots, p_{t-1} \in T(A)$ , as well as numbers  $q_1, q_2, \dots, q_{t-1}$  (not obligatory different), such that  $q_1 \in \overline{T(A)}, a_{p_1 q_1} \neq 0$  and  $q_i \in \overline{T(A)} \cup \{p_0, p_2, \dots, p_{i-1}\}, a_{p_i q_i} \neq 0$ , for every  $i \in \{2, 3, \dots, t-1\}$ . By using induction, we shall prove that for every  $n \in \{1, 2, \dots, t-1\}$ , there exists a path in  $G(A)$  from  $p_n$  to some  $j \in \overline{T(A)}$ . If  $n = 1$ , the statement is true for  $j = q_1 \in \overline{T(A)}$ . Let us now assume that it is true for all  $i \in \{1, 2, \dots, n-1\}$ , where  $n \leq t-1$ , and let us prove that it is then true for  $n$  also. We know that there exists  $q_n \in \overline{T(A)} \cup \{p_1, p_2, \dots, p_{n-1}\}$  such that  $a_{p_n q_n} \neq 0$ . If  $q_n \in \overline{T(A)}$ , then we can take  $j = q_n$ , else  $q_n = p_i$  for some  $i \in \{1, 2, \dots, n-1\}$ . Since by inductive hypothesis there exists a path in  $G(A)$  from  $p_i$  to some  $j \in \overline{T(A)}$ , then there also exists a path from  $p_n$  to that  $j \in \overline{T(A)}$ . Let  $T(A) \setminus \{p_1, p_2, \dots, p_{t-1}\} = \{i\}$ . Since  $a_{ii} \neq 0$  and  $r_i(A) = |a_{ii}| > 0$ , there exists  $k \in N \setminus \{i\}$  such that  $a_{ik} \neq 0$ . If  $k \in \overline{T(A)}$  then we have a path from  $i$  to  $k \in \overline{T(A)}$ , else  $k \in T(A) \setminus \{i\} = \{p_1, p_2, \dots, p_{t-1}\}$ , i.e.  $k = p_l$  for some  $l \in \{1, 2, \dots, t-1\}$ . Since we have proven that there exists a path in  $G(A)$  from  $p_l$  to some  $j \in \overline{T(A)}$ , then there also exists a path from  $i$  to that  $j \in \overline{T(A)}$ . Hence, we have proven that for every  $i \in T(A)$ , there exists a path in  $G(A)$  to some  $j \in \overline{T(A)}$ .  $\square$

The class of  $\mathcal{S}$ -SDD matrices is the class of H-matrices introduced independently by Gao and Wang in 1992 ([6]), and by Cvetković, Kostic and Varga in 2004 ([7, 2]). We use notation from [7, 2].

**Definition 2** *Given any matrix  $A = [a_{ij}] \in \mathbb{C}^{n \times n}, n \geq 2$ , and given any nonempty proper subset  $S$  of  $N$ , then  $A$  is an  $S$ -strictly diagonally dominant ( $S$ -SDD) if*

$$(1) \quad \begin{cases} i) |a_{ii}| > r_i^S(A) \text{ for all } i \in S, \\ ii) (|a_{ii}| - r_i^S(A))(|a_{jj}| - r_j^{\overline{S}}(A)) > r_i^{\overline{S}}(A)r_j^S(A) \text{ for all } i \in S, j \in \overline{S}. \end{cases}$$

We say that a matrix  $A \in \mathbb{C}^{n \times n}, n \geq 2$ , is  $\mathcal{S}$ -SDD, if there exists a nonempty proper subset  $S$  of  $N$ , such that  $A$  is  $S$ -SDD.

It can be shown that the intersection of classes of DD and  $\mathcal{S}$ -SDD matrices has a very simple characterization. Namely, let  $A = [a_{ij}] \in \mathbb{C}^{n \times n}, n \geq 2$ , be a DD matrix. Then,  $A$  is an  $\mathcal{S}$ -SDD if and only if  $T(A) = \emptyset$ , or  $A|_{T(A)^2}$  is an SDD matrix. Therefore, using  $\mathcal{S}$ -SDD matrices we conclude that if a matrix  $A$  is DD, such that  $T(A) = \emptyset$ , or  $A|_{T(A)^2}$  is an SDD matrix, then  $A$  is an H-matrix. We strengthen this result in Theorem 7.

Given any  $A \in \mathbb{C}^{n \times n}$ , let  $\mathcal{M}(A) = [\alpha_{ij}] \in \mathbb{R}^{n \times n}$  denote its comparison matrix, i.e.

$$\alpha_{ii} := |a_{ii}|, \text{ for all } i \in N,$$

$$\alpha_{ij} := -|a_{ij}|, \text{ for all } i, j \in N, i \neq j.$$

Let  $A|_{S^2}$  denote the principal submatrix of the matrix  $A$ , which corresponds to the set  $S$  of indices.

**Definition 3** *Given any matrix  $A = [a_{ij}] \in \mathbb{C}^{n \times n}$ ,  $n \geq 2$ , and given any nonempty proper subset  $S$  of  $N$ , ( $S = \{i_1, i_2, \dots, i_k\}$ ), then  $A$  is an  $S$ -H matrix if*

$$(2) \begin{cases} i) A|_{S^2} \text{ is an H-matrix,} \\ ii) \|\mathcal{M}^{-1}(A|_{S^2}) \cdot \mathbf{r}^{\overline{S}}(A)\|_{\infty} < B_2^S := \min_{j \in \overline{S}} \frac{|a_{jj}| - r_j^{\overline{S}}(A)}{r_j^{\overline{S}}(A)}, \end{cases}$$

where  $\mathbf{r}^{\overline{S}}(A) := [r_{i_1}^{\overline{S}}(A) \ r_{i_2}^{\overline{S}}(A) \ \dots \ r_{i_k}^{\overline{S}}(A)]^T$ ,  $\frac{a}{0} := \pm\infty$  (depending on the sign of  $a \neq 0$ ) and  $\frac{0}{0} := 0$ .

We say that a matrix  $A \in \mathbb{C}^{n \times n}$ ,  $n \geq 2$ , is an  $\mathcal{S}$ -H matrix, if there exists  $S \in \mathcal{P}(N) \setminus \{\emptyset, N\}$  such that  $A$  is an  $S$ -H matrix.

The following result is proven by Huang ([4]).

**Theorem 4** *Let  $A$  be an  $\mathcal{S}$ -H matrix. Then  $A$  is an H-matrix.*

In the same paper, Huang also gave the following characterization of diagonally dominant H-matrices.

**Theorem 5** *Let  $A = [a_{ij}] \in \mathbb{C}^{n \times n}$ ,  $n \geq 2$ , be a DD matrix. Then  $A$  is an H-matrix if and only if  $T(A) = \emptyset$ , or  $A$  is a  $T(A)$ -H matrix, i.e.*

$$(3) \begin{cases} i) A|_{T(A)^2} \text{ is an H-matrix,} \\ ii) \|\mathcal{M}^{-1}(A|_{T(A)^2}) \cdot \mathbf{r}^{\overline{T(A)}}(A)\|_{\infty} < B_2^{T(A)} := \min_{j \in \overline{T(A)}} \frac{|a_{jj}| - r_j^{\overline{T(A)}}(A)}{r_j^{\overline{T(A)}}(A)}, \end{cases}$$

We shall simplify that characterization by showing that the condition (3 ii) is surplus. We shall need the following well-known nonsingularity result of Taussky from 1949 ([5]), which is the special case of Theorem 1 or 2.

**Theorem 6** *Let  $A = [a_{ij}] \in \mathbb{C}^{n \times n}$ ,  $n \geq 2$ , be an irreducible DD+ matrix. Then  $A$  is nonsingular (and also an H-matrix).*

Let us first prove the special case of our statement, because we shall use it in the proof of the general case.

**Lemma 1** *Let  $A = [a_{ij}] \in \mathbb{C}^{n \times n}$ ,  $n \geq 2$ , be a DD matrix such that  $T(A) \neq \emptyset$ . If  $A|_{T(A)^2}$  is SDD by columns, then  $A$  is an H-matrix.*

*Proof.* We first conclude that  $A$  has to be a DD+ matrix. If  $A$  is irreducible, then from Theorem 6 we conclude that it is an H-matrix. If it is reducible, then there exists a permutation matrix  $P$ , such that  $F = PAP^T$  is the Frobenius normal form of the matrix  $A$  (see [2])

$$F = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1m} \\ 0 & R_{22} & \cdots & R_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_{mm} \end{bmatrix},$$

where each matrix  $R_{jj}, j \in \{1, 2, \dots, m\}$  is either a  $1 \times 1$  matrix, or an  $n_j \times n_j$  irreducible matrix with  $n_j \geq 2$ . If  $R_{jj} = [a_{kk}]$  for some  $j \in \{1, 2, \dots, m\}$  and  $k \in N$ , then  $a_{kk} \neq 0$ , because  $A|_{T(A)^2}$  is SDD by columns, which implies that  $A$  has nonzero diagonal entries. If  $R_{jj}$  is an  $n_j \times n_j$  irreducible matrix with  $n_j \geq 2$ , for some  $j \in \{1, 2, \dots, m\}$ , then  $R_{jj} = A|_{N_j^2}$  for some  $N_j \subset N$  such that  $|N_j| = n_j$ . Since  $A$  is DD,  $R_{jj}$  is also DD, and let us assume that it is not DD+. Then  $N_j \subseteq T(A)$ , which implies that  $R_{jj}$  is SDD by columns. A contradiction with the fact that it is DD which is not DD+. Hence, we have that  $R_{jj}$  is DD+. Now from Theorem 6 we conclude that  $R_{jj}$  is an H-matrix. We have concluded that  $R_{jj}$  is an H-matrix for every  $j \in \{1, 2, \dots, m\}$ . Therefore, there exist diagonal matrices  $D_j > 0$  such that  $R_{jj}D_j$  is an SDD matrix for every  $j \in \{1, 2, \dots, m\}$ . Since diagonal matrices  $c_j D_j$  have the same property for arbitrary positive real numbers  $c_j, j \in \{1, 2, \dots, m\}$ , we can easily construct a diagonal matrix  $D > 0$  such that  $FD$  is an SDD matrix, i.e.  $F$  is an H-matrix, or equivalently  $A$  is an H-matrix.  $\square$

The next theorem contains simple characterization of diagonally dominant H-matrices.

**Theorem 7** *Let  $A = [a_{ij}] \in \mathbb{C}^{n \times n}, n \geq 2$ , be a DD matrix. Then  $A$  is an H-matrix if and only if  $T(A) = \emptyset$ , or  $A|_{T(A)^2}$  is an H-matrix.*

*Proof.* Let us assume that  $A$  is an H-matrix and that  $T(A) \neq \emptyset$ . Then there exists a diagonal matrix  $D > 0$  such that  $AD$  is an SDD matrix. Then  $(AD)|_{T(A)^2} = A|_{T(A)^2}D|_{T(A)^2}$  is also an SDD matrix as principal submatrix of an SDD matrix. Hence,  $A|_{T(A)^2}$  is an H-matrix. If  $T(A) = \emptyset$ ,  $A$  is SDD and therefore an H-matrix. So let us assume that  $T(A) \neq \emptyset$ , and that  $A|_{T(A)^2}$  is an H-matrix. Then  $A$  has to be DD+ because H-matrices have at least one SDD row. Also,  $A|_{T(A)^2}^T$  is an H-matrix, therefore there exists a diagonal matrix  $D_1 > 0$  such that  $A|_{T(A)^2}^T D_1$  is an SDD matrix. Let  $D > 0$  be a diagonal matrix such that  $D|_{T(A)^2} = D_1$  and  $D|_{\overline{T(A)}^2} = \text{diag}(1, 1, \dots, 1)$ . With  $B = DA$ , we have that  $B$  is DD+,  $T(A) = T(B)$  and  $B|_{T(B)^2}$  is SDD by columns. From Lemma 1, we conclude that  $B$  is an H-matrix, or equivalently  $A$  is an H-matrix.  $\square$

Previous theorem gives us practical algorithm for checking whether a given diagonally dominant matrix is an H-matrix or not.

**Corollary 1** *Let  $A = [a_{ij}] \in \mathbb{C}^{n \times n}, n \geq 2$ , be a DD matrix. Then  $A$  is not an H-matrix if and only if there exists  $M \subseteq T(A)$ , such that  $A|_{M^2}$ , which is DD, is not a DD+ matrix.*

*Proof.* Let us assume that there exists  $M \subseteq T(A)$ , such that  $A|_{M^2}$ , which is DD, is not a DD+ matrix. Since every H-matrix has at least one SDD row,  $A|_{M^2}$  is not an H-matrix.

Since every principal submatrix of an H-matrix is again an H-matrix, we conclude that  $A$  is not an H-matrix. Let us now assume that  $A$  is not an H-matrix. If  $A$  is not DD+ then  $M = T(A) = N$ , else from Theorem 7 it follows that  $A_1 = A|_{T(A)^2}$  is not an H-matrix. For the sake of the simplicity of the proof, let us take indices of elements of the submatrix  $A|_{T(A)^2}$  the same as they were in the matrix  $A$ , i.e. they are all from  $T(A)$ . If  $A_1$  is not DD+ then  $M = T(A)$ , else it follows that  $A_2 = A_1|_{T(A_1)^2} = A|_{T(A_1)^2}$  is not an H-matrix. By continuing this procedure, we get in a finite number of steps that  $A|_{M^2}$  is not DD+ for some  $M \subseteq T(A)$ ,  $|M| \geq 2$  else we finish with  $1 \times 1$  matrix  $A|_{T(A_k)^2}$ , which is not an H-matrix, i.e.  $A|_{T(A_k)^2} = [0]$ . In that case, we take  $M = T(A_k)$ .  $\square$

Finally, by recursively applying Theorem 7, we prove that the Shivakumar-Chew-Farid sufficient condition for a diagonally dominant matrix to be an H-matrix is also necessary.

**Theorem 8** *Let  $A = [a_{ij}] \in \mathbb{C}^{n \times n}$ ,  $n \geq 2$ , be a DD matrix. Then  $A$  is an H-matrix if and only if  $A$  has nonzero diagonal entries and  $T(A)$  is an interwoven set of indices for  $A$  (equivalently,  $T(A) = \emptyset$  or for each  $i_0 \in T(A)$  there exists a nonzero elements chain of the form  $a_{i_0 i_1}, a_{i_1 i_2}, \dots, a_{i_{r-1} i_r}$ , with  $i_r \in \overline{T(A)}$ ).*

*Proof.* The reverse direction follows from Theorem 2. Let us assume that  $A$  is a diagonally dominant H-matrix such that  $|T(A)| > 1$ . Since  $A$  is an H-matrix, it has nonzero diagonal entries. For the sake of the simplicity of the proof, let us take indices of elements of the submatrix  $A|_{T(A)^2}$  the same as they were in the matrix  $A$ , i.e. they are all from  $T(A)$ . It follows from Theorem 7 that  $A_1 = A|_{T(A)^2}$  is an H-matrix. If  $|T(A_1)| \leq 1$ , then we can take for  $p_1, p_2, \dots, p_{t-1}$ , where  $t = |T(A)|$ , some  $t-1$  different numbers from  $T(A) \setminus T(A_1)$ . Then for each such  $p_i$ , there exists  $q_i \in \overline{T(A)}$  such that  $a_{p_i q_i} \neq 0$ ,  $i \in \{1, 2, \dots, t-1\}$ . If  $|T(A_1)| > 1$ , then we choose  $p_1, p_2, \dots, p_{k_1} \in T(A)$ , such that  $T(A) \setminus T(A_1) = \{p_1, p_2, \dots, p_{k_1}\}$ . For each such  $p_i$ , there exists  $q_i \in \overline{T(A)}$ , such that  $a_{p_i q_i} \neq 0$ ,  $i \in \{1, 2, \dots, k_1\}$ . From Theorem 7 it follows that  $A_2 = A_1|_{T(A_1)^2}$  is an H-matrix. If  $|T(A_2)| \leq 1$ , then we can take for  $p_{k_1+1}, p_{k_1+2}, \dots, p_{t-1}$ , some  $t - k_1 - 1$  different numbers from  $T(A_1) \setminus T(A_2)$ . Then for each such  $p_i$ , there exists  $q_i \in \{p_1, p_2, \dots, p_{k_1}\}$ , such that  $a_{p_i q_i} \neq 0$ ,  $i \in \{k_1 + 1, k_1 + 2, \dots, t-1\}$ . If  $|T(A_2)| > 1$ , then we choose  $p_{k_1+1}, p_{k_1+2}, \dots, p_{k_2} \in T(A)$ , such that  $T(A_1) \setminus T(A_2) = \{p_{k_1+1}, p_{k_1+2}, \dots, p_{k_2}\}$ . For each such  $p_i$ , there exists  $q_i \in \{p_1, p_2, \dots, p_{k_1}\}$ , such that  $a_{p_i q_i} \neq 0$ ,  $i \in \{k_1 + 1, k_1 + 2, \dots, k_2\}$ . We continue this procedure. Since  $\{|T(A_i)|\}_i$  is decreasing sequence of natural numbers, after finite number of steps, we get that  $|T(A_m)| \leq 1$ . Then we take for  $p_{k_{m-1}+1}, p_{k_{m-1}+2}, \dots, p_{t-1}$ , some  $t - k_{m-1} - 1$  different numbers from  $T(A_{m-1}) \setminus T(A_m)$ . Then for each such  $p_i$ , there exists  $q_i \in \{p_{k_{m-2}+1}, p_{k_{m-2}+2}, \dots, p_{k_{m-1}}\}$ , such that  $a_{p_i q_i} \neq 0$ ,  $i \in \{k_{m-1} + 1, k_{m-1} + 2, \dots, t-1\}$ . Thus, we have constructed different numbers  $p_1, p_2, \dots, p_{t-1} \in T(A)$ , as well as numbers  $q_1, q_2, \dots, q_{t-1}$  (not obligatory different), such that  $q_1 \in \overline{T(A)}$ ,  $a_{p_1 q_1} \neq 0$  and  $q_i \in \overline{T(A)} \cup \{p_1, p_2, \dots, p_{i-1}\}$ ,  $a_{p_i q_i} \neq 0$ , for every  $i \in \{2, 3, \dots, t-1\}$ . Hence,  $T(A)$  is an interwoven set of indices for the matrix  $A$ .  $\square$

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